

(%i1)

```
/* define special summation function */
f(i,j) := sum(R[i,j,sigma,0]*gContr[i,sigma]*gContr[j,0],sigma,0,3)
        + sum(R[i,j,sigma,1]*gContr[i,sigma]*gContr[j,1],sigma,0,3)
        + sum(R[i,j,sigma,2]*gContr[i,sigma]*gContr[j,2],sigma,0,3)
        + sum(R[i,j,sigma,3]*gContr[i,sigma]*gContr[j,3],sigma,0,3);
```

(%o1) $f(i, j) := \text{sum}(R_{i, j, \sigma, 0} g_{\text{Contr } i, \sigma} g_{\text{Contr } j, 0, \sigma, 0, 3}) +$

$\text{sum}(R_{i, j, \sigma, 1} g_{\text{Contr } i, \sigma} g_{\text{Contr } j, 1, \sigma, 0, 3}) +$

$\text{sum}(R_{i, j, \sigma, 2} g_{\text{Contr } i, \sigma} g_{\text{Contr } j, 2, \sigma, 0, 3}) +$

$\text{sum}(R_{i, j, \sigma, 3} g_{\text{Contr } i, \sigma} g_{\text{Contr } j, 3, \sigma, 0, 3})$

(%i2) /* define coordinate vector */

```
array(x, 3);
[x[0],x[1],x[2],x[3]]: [t, x1, x2, x3];
```

(%o2) x

(%o3) [t , x1 , x2 , x3]

(%i4) /* g1 is symm. metric with indices 1...4 */

```
g1: matrix(
  [-1,0,0,0],
  [0,t^(2*p1),0,0],
  [0,0,t^(2*p2),0],
  [0,0,0,t^(2*p3)]
);
```

(%o4)
$$\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & t^{2 p1} & 0 & 0 \\ 0 & 0 & t^{2 p2} & 0 \\ 0 & 0 & 0 & t^{2 p3} \end{bmatrix}$$

(%i5) /* contravariant g is inverse of g */

```
gContr1: ratsimp(invert(g1));
```

(%o5)
$$\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & \frac{1}{t^{2 p1}} & 0 & 0 \\ 0 & 0 & \frac{1}{t^{2 p2}} & 0 \\ 0 & 0 & 0 & \frac{1}{t^{2 p3}} \end{bmatrix}$$

(%i6)

```

/* g1 and gContr1 are transformed to g and gContr (indices 0...3) */
for mu:0 thru 3 do {
for nu:0 thru 3 do {
    g      [mu,nu]: g1      [mu+1, nu+1],
    gContr[mu,nu]: gContr1[mu+1, nu+1]
}}$

```

```

(%i7) /* computation of Christoffel symbols Gamma^sigma_mu_nu */
for sigma:0 thru 3 do {
for mu:0 thru 3 do {
for nu:0 thru 3 do {
    Gamma[sigma,mu,nu] :
    /* rho sum by function call: */
    sum(
        1/2 * gContr[sigma,rho]*(
            diff(g[nu,rho],x[mu] ) +
            diff(g[rho,mu],x[nu] ) -
            diff(g[mu,nu] ,x[rho])),
        rho, 0, 3),
    /* evaluate differentiation dy/dr */
    Gamma[sigma,mu,nu]: ev(Gamma[sigma,mu,nu],diff)
}}}$

```

```

(%i8) /* display Gamma's being different from zero */
for i:0 thru 3 do {
for j:0 thru 3 do {
for k:0 thru 3 do {
    if Gamma[i,j,k] # 0 then {
        display(Gamma[i,j,k])
    }}}$

```

$$\Gamma_{0,1,1} = p1 \ t^{2p1-1}$$

$$\Gamma_{0,2,2} = p2 \ t^{2p2-1}$$

$$\Gamma_{0,3,3} = p3 \ t^{2p3-1}$$

$$\Gamma_{1,0,1} = \frac{p1}{t}$$

$$\Gamma_{1,1,0} = \frac{p1}{t}$$

$$\Gamma_{2,0,2} = \frac{p2}{t}$$

$$\Gamma_{2,2,0} = \frac{p2}{t}$$

$$\Gamma_{3,0,3} = \frac{p3}{t}$$

$$\Gamma_{3,3,0} = \frac{p3}{t}$$

```
(%i9) /* compute Riemann tensor elements */
for rho:0 thru 3 do {
for sigma:0 thru 3 do {
for mu:0 thru 3 do {
for nu:0 thru 3 do {
R[rho,sigma,mu,nu] :
diff(Gamma[rho,nu,sigma],x[mu]) -
diff(Gamma[rho,mu,sigma],x[nu]) +
/* lambda sums by function call: */
sum(
Gamma[rho,mu,lambda] * Gamma[lambda,nu,sigma] -
Gamma[rho,nu,lambda] * Gamma[lambda,mu,sigma],
lambda, 0, 3)
}}}}$
```

```
(%i10) /* display R's being different from zero */
for i:0 thru 3 do {
for j:0 thru 3 do {
for k:0 thru 3 do {
for l:0 thru 3 do {
R[i,j,k,l] : /*ratsimp*/(factor(R[i,j,k,l])),
if R[i,j,k,l] # 0 then display(R[i,j,k,l])
}}}}$
```

$$R_{0,1,0,1} = (p1 - 1) p1 t^{2 p1 - 2}$$

$$R_{0,1,1,0} = - (p1 - 1) p1 t^{2 p1 - 2}$$

$$R_{0,2,0,2} = (p2 - 1) p2 t^{2 p2 - 2}$$

$$R_{0,2,2,0} = - (p2 - 1) p2 t^{2 p2 - 2}$$

$$R_{0,3,0,3} = (p3 - 1) p3 t^{2 p3 - 2}$$

$$R_{0,3,3,0} = - (p3 - 1) p3 t^{2 p3 - 2}$$

$$R_{1,0,0,1} = \frac{(p1 - 1) p1}{t^2}$$

$$R_{1,0,1,0} = - \frac{(p1 - 1) p1}{t^2}$$

$$R_{1,2,1,2} = p1 p2 t^{2 p2 - 2}$$

$$R_{1,2,2,1} = - p1 p2 t^{2 p2 - 2}$$

$$R_{1,3,1,3} = p1 p3 t^{2 p3 - 2}$$

$$R_{1,3,3,1} = - p1 p3 t^{2 p3 - 2}$$

$$R_{2,0,0,2} = \frac{(p2 - 1) p2}{t^2}$$

$$R_{2,0,2,0} = - \frac{(p2 - 1) p2}{t^2}$$

$$R_{2,1,1,2} = -p_1 p_2 t^{2p_1-2}$$

$$R_{2,1,2,1} = p_1 p_2 t^{2p_1-2}$$

$$R_{2,3,2,3} = p_2 p_3 t^{2p_3-2}$$

$$R_{2,3,3,2} = -p_2 p_3 t^{2p_3-2}$$

$$R_{3,0,0,3} = \frac{(p_3-1)p_3}{t^2}$$

$$R_{3,0,3,0} = -\frac{(p_3-1)p_3}{t^2}$$

$$R_{3,1,1,3} = -p_1 p_3 t^{2p_1-2}$$

$$R_{3,1,3,1} = p_1 p_3 t^{2p_1-2}$$

$$R_{3,2,2,3} = -p_2 p_3 t^{2p_2-2}$$

$$R_{3,2,3,2} = p_2 p_3 t^{2p_2-2}$$

```
(%i11) /* Ricci tensor Ric[mu,nu] */
for mu:0 thru 3 do {
for nu:0 thru 3 do {
Ric[mu,nu]: sum(R[lambda,mu,lambda,nu], lambda, 0, 3)
}}$
```

```
(%i12) /* display Ric's being different from zero */
for i:0 thru 3 do {
for j:0 thru 3 do {
Ric[i,j] : /*ratsimp*/(factor(Ric[i,j])),
if Ric[i,j] # 0 then display(Ric[i,j])
}}$
```

$$Ric_{0,0} = -\frac{p_3^2 - p_3 + p_2^2 - p_2 + p_1^2 - p_1}{t^2}$$

$$Ric_{1,1} = p_1 (p_3 + p_2 + p_1 - 1) t^{2p_1-2}$$

$$Ric_{2,2} = p_2 (p_3 + p_2 + p_1 - 1) t^{2p_2-2}$$

$$Ric_{3,3} = p_3 (p_3 + p_2 + p_1 - 1) t^{2p_3-2}$$

```
(%i13) /* Ricci Scalar */
RicSc: sum(gContr[0,lambda]*Ric[lambda,0], lambda, 0, 3)
+ sum(gContr[1,lambda]*Ric[lambda,1], lambda, 0, 3)
+ sum(gContr[2,lambda]*Ric[lambda,2], lambda, 0, 3)
+ sum(gContr[3,lambda]*Ric[lambda,3], lambda, 0, 3)
;
```

```
(%o13) 
$$\frac{p_3^2 - p_3 + p_2^2 - p_2 + p_1^2 - p_1}{t^2} + \frac{p_3(p_3 + p_2 + p_1 - 1)}{t^2} + \frac{p_2(p_3 + p_2 + p_1 - 1)}{t^2} +$$

```

$$\frac{p1(p3 + p2 + p1 - 1)}{t^2}$$

```
(%i14) ratsimp(RicSc);
```

```
(%o14) 
$$\frac{2 p3^2 + (2 p2 + 2 p1 - 2) p3 + 2 p2^2 + (2 p1 - 2) p2 + 2 p1^2 - 2 p1}{t^2}$$

```

```
(%i15)
```

```
/* Test for R^q */
for mu: 0 thru 3 do (
for sigma:0 thru 3 do (
for nu: 0 thru 3 do (
for rho: 0 thru 3 do (
  R_q: R[mu,sigma,nu,rho] + R[mu,rho,sigma,nu] + R[mu,nu,rho,sigma],
  if R_q # 0 then (
    display("====Einstein equation R^q=0 not fulfilled! "),
    display(mu,sigma,nu,rho),
    display(R_q)
  )
)))));
(%o15) done
```

```
(%i16) /* Raising of indices,
        contravarinat metric el. is g^x^x(contr.) = 1/g_x_x(cov.) */
/*print("Riemann elements R^0_1^0^1, R^0_2^0^2, R^0_3^0^3:");*/

R0101: f(0,1);
R0202: f(0,2);
R0303: f(0,3);
```

```
(%o16) 
$$-\frac{(p1 - 1)p1}{t^2}$$

```

```
(%o17) 
$$-\frac{(p2 - 1)p2}{t^2}$$

```

```
(%o18) 
$$-\frac{(p3 - 1)p3}{t^2}$$

```

```
(%i19) R0101: factor(R0101);
        R0202: factor(R0202);
        R0303: factor(R0303);
```

```
(%o19) 
$$-\frac{(p1 - 1)p1}{t^2}$$

```

```
(%o20) 
$$-\frac{(p2 - 1)p2}{t^2}$$

```

```
(%o21) 
$$-\frac{(p3 - 1)p3}{t^2}$$

```

```
(%i22) R1010: f(1,0);  
       R1212: f(1,2);  
       R1313: f(1,3);
```

```
(%o22) (p1 - 1) p1 t-2 p1 - 2
```

```
(%o23) p1 p2 t-2 p1 - 2
```

```
(%o24) p1 p3 t-2 p1 - 2
```

```
(%i25) R1010: factor(R1010);  
       R1212: factor(R1212);  
       R1313: factor(R1313);
```

```
(%o25) (p1 - 1) p1 t-2 p1 - 2
```

```
(%o26) p1 p2 t-2 p1 - 2
```

```
(%o27) p1 p3 t-2 p1 - 2
```

```
(%i28) R2020: f(2,0);  
       R2121: f(2,1);  
       R2323: f(2,3);
```

```
(%o28) (p2 - 1) p2 t-2 p2 - 2
```

```
(%o29) p1 p2 t-2 p2 - 2
```

```
(%o30) p2 p3 t-2 p2 - 2
```

```
(%i31) R2020: factor(R2020);  
       R2121: factor(R2121);  
       R2323: factor(R2323);
```

```
(%o31) (p2 - 1) p2 t-2 p2 - 2
```

```
(%o32) p1 p2 t-2 p2 - 2
```

```
(%o33) p2 p3 t-2 p2 - 2
```

```
(%i34) R3030: f(3,0);  
       R3131: f(3,1);  
       R3232: f(3,2);
```

```
(%o34) (p3 - 1) p3 t-2 p3 - 2
```

```
(%o35) p1 p3 t-2 p3 - 2
```

```
(%o36) p2 p3 t-2 p3 - 2
```

```
(%i37) R3030: factor(R3030);  
       R3131: factor(R3131);  
       R3232: factor(R3232);
```

```
(%o37) (p3 - 1) p3 t-2 p3 - 2
```

```
(%o38)  $p_1 p_3 t^{-2 p_3 - 2}$ 
```

```
(%o39)  $p_2 p_3 t^{-2 p_3 - 2}$ 
```

```
(%i40) /* Coulomb law */
DivE : R0101 + R0202 + R0303;
```

```
(%o40) 
$$-\frac{(p_3 - 1)p_3}{t^2} - \frac{(p_2 - 1)p_2}{t^2} - \frac{(p_1 - 1)p_1}{t^2}$$

```

```
(%i41) ratsimp(DivE);
```

```
(%o41) 
$$-\frac{p_3^2 - p_3 + p_2^2 - p_2 + p_1^2 - p_1}{t^2}$$

```

```
(%i42) /* J[r] */
Jr : -(R1010 + R1212 + R1313);
```

```
(%o42) 
$$-p_1 p_3 t^{-2 p_1 - 2} - p_1 p_2 t^{-2 p_1 - 2} - (p_1 - 1) p_1 t^{-2 p_1 - 2}$$

```

```
(%i43) ratsimp(Jr);
```

```
(%o43) 
$$-(p_1 p_3 + p_1 p_2 + p_1^2 - p_1) t^{-2 p_1 - 2}$$

```

```
(%i44) /* J[theta] */
Jtheta : -(R2020 + R2121 + R2323);
```

```
(%o44) 
$$-p_2 p_3 t^{-2 p_2 - 2} - (p_2 - 1) p_2 t^{-2 p_2 - 2} - p_1 p_2 t^{-2 p_2 - 2}$$

```

```
(%i45) ratsimp(Jtheta);
```

```
(%o45) 
$$-(p_2 p_3 + p_2^2 + (p_1 - 1) p_2) t^{-2 p_2 - 2}$$

```

```
(%i46) /* J[phi] */
Jphi : -(R3030 + R3131 + R3232);
```

```
(%o46) 
$$-(p_3 - 1) p_3 t^{-2 p_3 - 2} - p_2 p_3 t^{-2 p_3 - 2} - p_1 p_3 t^{-2 p_3 - 2}$$

```

```
(%i47) ev(ratsimp(Jphi),r);
```

```
(%o47) 
$$-(p_3^2 + (p_2 + p_1 - 1) p_3) t^{-2 p_3 - 2}$$

```

```
(%i50) DivE_p: ev(at(DivE, [p1=sqrt(2), p2=sqrt(2), p3=0]));
```

```
(%o50) 
$$-\frac{2(\sqrt{2} - 1)\sqrt{2}}{t^2}$$

```

```
(%i51) DivE_p: ev(at(DivE, [p1=1, p2=-1, p3=0]));
```

```
(%o51)  -  $\frac{2}{t^2}$ 
```

```
(%i52)  J1_p: ev(at(Jr, [p1=1,p2=-1,p3=0]));
```

```
(%o52)   $\frac{1}{t^4}$ 
```

```
(%i53)  J2_p: ev(at(Jtheta, [p1=1,p2=-1,p3=0]));
```

```
(%o53)  - 1
```

```
(%i54)  J3_p: ev(at(Jphi, [p1=1,p2=-1,p3=0]));
```

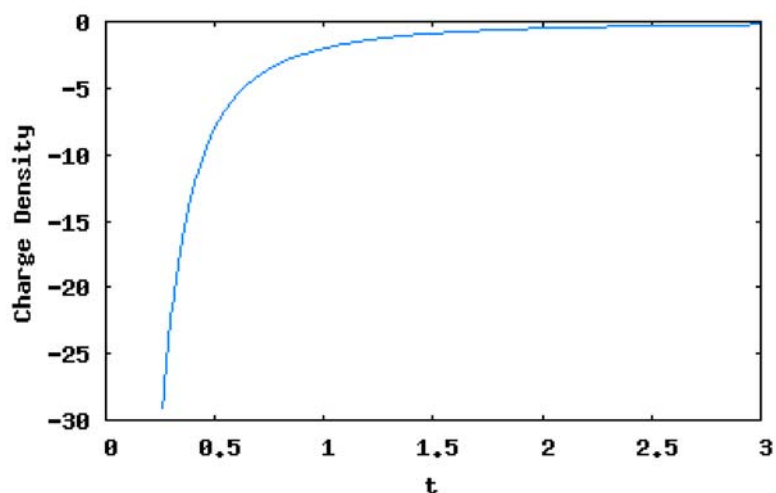
```
(%o54)  0
```

```
(%i56)
```

```
wxplot2d([DivE_p], [t,0,3],[y,-30,0], [gnuplot_preamble, "set zeroaxis;"],
[xlabel, "t"], [ylabel, "Charge Density"])$
```

Output file "C:/Documents and Settings/Administrator/maxout.png".

```
(%t56)
```

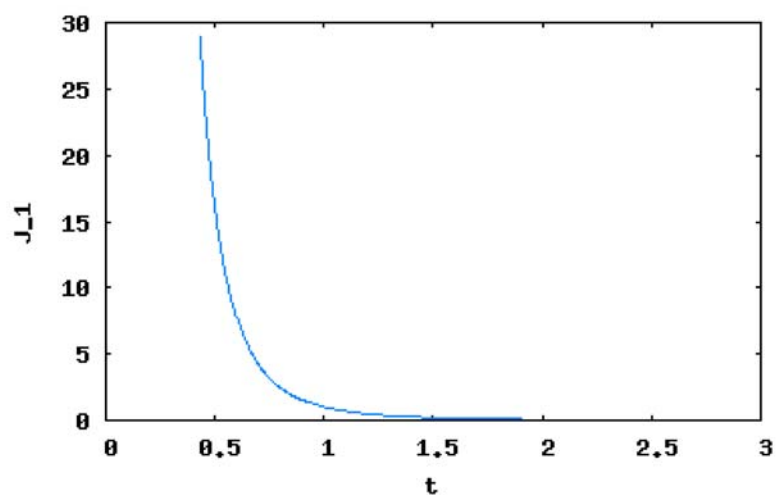


```
(%i64)
```

```
wxplot2d([J1_p], [t,0,3],[y,0,30], [gnuplot_preamble, "set zeroaxis;"],
[xlabel, "t"], [ylabel, "J_1"])$
```

Output file "C:/Documents and Settings/Administrator/maxout.png".

(%t64)

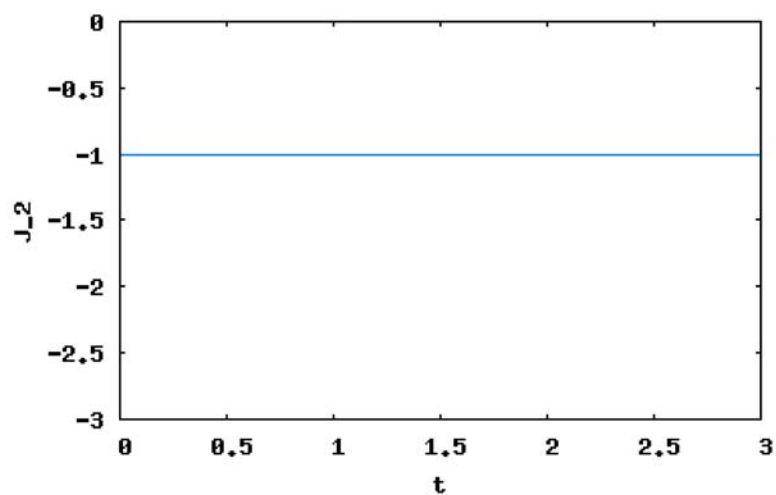


(%i60)

```
wxplot2d([J2_p], [r,0,3],[y,-3,0], [gnuplot_preamble, "set zeroaxis;"],
[xlabel, "t"], [ylabel, "J_2"])$
```

Output file "C:/Documents and Settings/Administrator/maxout.png".

(%t60)

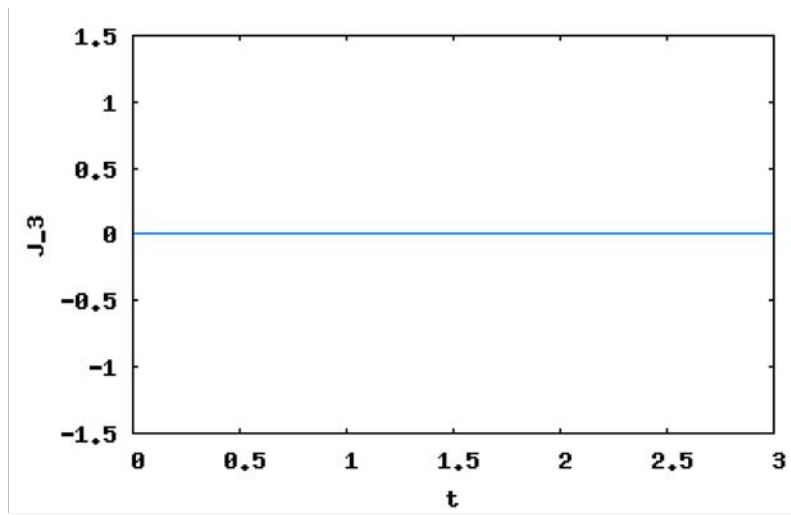


(%i63)

```
wxplot2d([J3_p], [r,0,3],[y,-1.5,1.5], [gnuplot_preamble, "set zeroaxis;"],
[xlabel, "t"], [ylabel, "J_3"])$
```

Output file "C:/Documents and Settings/Administrator/maxout.png".

(%t63)



(%i65)