

230(1) : Influence of Phonon Assumption 2 (ENR).

Ref. R. Eisberg and R. Resnick, "Quantum Physics",
(Wiley, 1985, 2nd. ed., p. 399)

Phonons are quanta of acoustic radiation and are emitted and absorbed by vibrating atoms in a solid lattice. They are conventionally massless

and:

$$E = \hbar \omega \quad - (1)$$

where ω is the angular frequency of the acoustic vibration. The phonon propagates through the crystal lattice, which contains a gas of phonons. The dispersion relation is:

$$\omega = v k \quad - (2)$$

where v is the speed of sound.

Now consider eq. (24) of UFT 229 in

the presence of a phonon:

$$T = \frac{4}{\left(2\theta + \frac{1}{2\theta}\right)^2} \quad - (3)$$

$$\theta = \exp \left(\frac{\left(\frac{2\mu}{\hbar}\right)^{1/2}}{2} \int_a^b \left(V(r) - E \right)^{1/2} dr \right)$$

with the potential function $V(r)$ defined as in

2) eq. (27) of UFT 229.

The absorption of a phonon means that:

$$E \rightarrow E + \hbar\omega \quad - (5)$$

$$E_1 = E_0 + \hbar\omega \quad - (6)$$

i.e

Resonant or phonon absorption occurs at:

$$\omega = (E_1 - E_0) / \hbar \quad - (7)$$

At ω frequency ω the transmission coefficient changes to:

$$T_1 = \frac{4}{\left(2\theta_1 + \frac{1}{2\theta_1}\right)^2} \quad - (8)$$

where:

$$\theta_1 = \exp \left(\frac{(2\mu)^{1/2}}{\hbar} \int_a^b \left(V(r) - E - \hbar\omega \right)^{1/2} dr \right) \quad - (9)$$

The wavelength of a phonon is about 10^{-10} m, and they are quantized normal modes of lattice vibration.

In n phonon absorption:

$$E \rightarrow E + n\hbar\omega \quad - (10)$$

where:

$$n = 1, 2, 3, \dots \quad (11)$$

so:

$$T_n = \frac{4}{\left(2\theta_n + \frac{1}{2\theta_n}\right)^2} \quad - (12)$$

where:

$$\theta_n = \frac{2\pi}{\hbar} \int_a^b \left(V(r) - (E + n\hbar\omega) \right)^{1/2} dr \quad - (13)$$

At \mathcal{Q} point:

$$E + n\hbar\omega = V(r) \quad - (14)$$

then

$$T_n = \frac{4}{2.5^2} = 0.64. \quad - (15)$$

This occurs at \mathcal{Q} point:

$$\boxed{\omega = \frac{V - E}{n\hbar}} \quad - (16)$$

If:

$$E \ll V \quad - (17)$$

then:

$$\omega \sim \frac{V}{n\hbar} \quad - (18)$$