

24(7): Development of the Theory of the Gravitational Red Shift.

The theory of the gravitational red shift is based on the metric:

$$c^2 d\tau^2 = (c^2 - v^2) dt^2 \quad - (1)$$

of special relativity, in which:

$$v^2 = \left(\frac{dr}{dt}\right)^2 + r^2 \left(\frac{d\theta}{dt}\right)^2 \quad - (2)$$

For a conical section:

$$r = \frac{d}{1 + \epsilon \cos \theta} \quad - (3)$$

it follows that:

$$v^2 = m\phi \left(\frac{2}{r} - \frac{1}{a} \right) \quad - (4)$$

where

$$\begin{aligned} \frac{1}{a} &= \frac{1 - \epsilon^2}{d} \quad - (5) \\ &= \frac{2V}{m\phi} \end{aligned}$$

where

$$V = -\frac{m\phi}{r} + \frac{L^2}{2mr^2} \quad - (6)$$

In the conventional theory of the gravitational red shift it is assumed that:

$$v^2 = \frac{2MG}{r} \quad (7)$$

So
$$\frac{dr}{dt} = \left(1 - \frac{2MG}{c^2 r}\right)^{1/2} = \left(1 - \frac{v^2}{c^2}\right)^{1/2} \quad (8)$$

The gravitational red shift can then be defined by:

$$1 - \frac{dr}{dt} \sim \frac{MG}{c^2 r} \quad (9)$$

if
$$r_s = \frac{2MG}{c^2} \ll r \quad (10)$$

Therefore r_s is the origin of the old Schwarzschild radius r_s .

It is based on the assumption:

$$\frac{1}{a} = \frac{1 - \epsilon^2}{2} = 0 \quad (11)$$

Eq. (11) means:

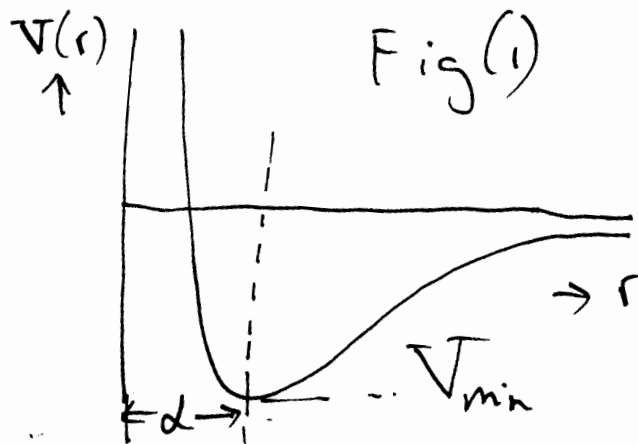
$$\epsilon = 1 \quad (12)$$

which is true for a circle:

$$\frac{dr}{dt} = 0 \quad (13)$$

So

$$\frac{d^2 r}{dt^2} = 0 \quad (14)$$



With reference to Fig (1) and previous work,
Eq. (14) occurs at

$$\begin{aligned} m \frac{d^2 r}{dt^2} &= - \frac{\partial V(r)}{\partial r} \\ &= - \frac{mM_G}{r^2} + \frac{L^2}{mr^3} \quad - (15) \\ &= 0 \end{aligned}$$

i.e.

$$r = d \quad - (16)$$

Eq. (15) defines a turning point, and the
minimum of the potential V.

In α theory the same turning point
defines the planetary precession, light deflection due
to gravitation and gravitational time delay.
The phenomena usually attributed to the old Schwarzschild
metric are all due to the deflection of the turning
point. In α theory, the turning point is the
same, i.e. the turning point of the orbit:

$$r = \frac{d}{1 + \epsilon \cos(\alpha\theta)} \quad - (17)$$

$$r = d \quad - (17a)$$

However in α theory:

$$\begin{aligned}
 m \frac{d^2 r}{dt^2} &= -\frac{dV(r)}{dr} = (x^2 - 1) \frac{\bar{L}^2}{mr^3} - \frac{x^2 L^2}{d m r^3} + \frac{L^2}{mr^3} \\
 &= x^2 \left(\frac{L^2}{mr^3} - \frac{L^2}{d m r^3} \right) \quad \text{--- (18)} \\
 &= 0
 \end{aligned}$$

To be rigorously self consistent, the velocity v in eq. (1) must be calculated with the precessing orbit of x theory:

$$r = \frac{d}{1 + \epsilon \cos(x\theta)} \quad \text{--- (19)}$$

and not with the Newtonian orbit:

$$r = \frac{d}{1 + \epsilon \cos \theta} \quad \text{--- (20)}$$

From eqs. (2) and (19):

$$\begin{aligned}
 v^2 &= \left(\frac{dr}{dt} \right)^2 + r^2 \left(\frac{d\theta}{dt} \right)^2 \\
 &= \omega^2 \left(r^2 + \left(\frac{dr}{d\theta} \right)^2 \right) \quad \text{--- (21)} \\
 &= \omega^2 \left(r^2 + \frac{x^2 \epsilon^2 r^4 \sin^2(x\theta)}{d^2} \right)
 \end{aligned}$$

where:

$$\cos(x\theta) = \frac{1}{\epsilon} \left(\frac{d}{r} - 1 \right) \quad - (7)$$

So:

$$v^2 = \omega^2 \left(r^2 + \left(\frac{x\epsilon r^2}{d} \right)^2 \left(1 - \frac{1}{\epsilon^2} \left(\frac{d}{r} - 1 \right)^2 \right) \right) \quad - (8)$$

Now use:

$$\omega = \frac{L}{mr^2} \quad - (9)$$

so

$$\begin{aligned} v^2 &= \frac{L^2}{m^2} \left(\frac{1}{r^2} + x^2 \left(\frac{\epsilon^2}{d^2} - \frac{1}{d^2} \left(\frac{d}{r} - 1 \right)^2 \right) \right) \\ &= \frac{L^2}{m^2} \left(\frac{1}{r^2} + x^2 \left(\frac{\epsilon^2}{d^2} - \frac{1}{d^2 r^2} (d^2 - 2rd + r^2) \right) \right) \\ &= \frac{L^2}{m^2} \left(\frac{2x^2}{dr} + x^2 \left(\frac{\epsilon^2 - 1}{d^2} \right) + \frac{1}{r^2} (1 - x^2) \right) \quad - (10) \end{aligned}$$

In the near Newtonian approximation:

$$d = \frac{L^2}{m^2 M G} \quad - (11)$$

so the rigorously correct velocity of the

processing orbit (17) is:

$$v^2 = MGx^2 \left(\frac{2}{r} - \frac{1}{a} \right) + \frac{L^2}{m^2 r^2} (1 - x^2) \quad - (12)$$

where

$$x = 1 + \frac{3MG}{c^2 a} \quad - (13)$$

It is clear therefore that the Schwarzschild radius is only an approximation. It is not consistent or correct to calculate an orbital precession from the old Schwarzschild metric:

$$c^2 d\tau^2 = c^2 dt^2 \left(1 - \frac{r_s}{r} \right) - dr^2 \left(1 - \frac{r_s}{r} \right)^{-1} - r^2 d\theta^2 \quad - (14)$$

which is incorrectly claimed to be a solution of an incorrect equation, the so called Einstein field equation.

The self inconsistency of eq. (14) is easily shown using the turning point method. From eq. (14) the turning point is:

$$r^2 - dr + r_0 d = 0 \quad - (15)$$

but the turning point from the rigorously correct

eq. (19) is:

$$x^2(d-r) = 0 \quad (16)$$

where x is observed to high experimental precision to be:

$$x = 1 + \frac{3MG}{c^2 d} \quad (17)$$

for all observable precessions in the universe.

It is not possible to force the incorrect eq. (15) to be the correct eq. (16). This procedure would mean that

$$r^2 - dr + r_0 d = ? x^2(d-r) \quad (18)$$

and

$$x^2 = ? \frac{r^2 - dr + r_0 d}{d-r} \quad (19)$$

i.e. x would depend on r , but experimentally it does not depend on r , QED

In fact, the orbit from the old S.M. (4) is

$$r = \frac{d}{1 + \epsilon \cos \left(\left(\frac{r^2 - dr + r_0 d}{d-r} \right)^{1/2} \theta \right)} \quad (20)$$

8) and in fact the old Einstein field equation produces the orbit (20), which is not a precessing ellipse, and certainly not the observed precessing ellipse.

The correct precessing ellipse is defined from eq. (17) by:

$$\theta = \frac{1}{x} \cos^{-1} \left(\frac{1}{\epsilon} \left(\frac{d}{r} - 1 \right) \right) - (21)$$

where

$$x = 1 + \frac{3MG}{c^2 d} - (22)$$

but the Einstein theory gives eq. (21) with:

$$x = \left(\frac{r^2 - dr + r_0 d}{d - r} \right)^{1/2} - (23)$$

Suggested Numerical Work

Plot eq. (21), i.e. θ versus r , with x given by eqs. (22) and (23), and compare.
