

315(1): Vector Form of the JCE Identity

The JCE identity is, for MFT 3B:

$$D_\rho R^{\kappa}_{\lambda\mu\nu} + D_\nu R^{\kappa}_{\lambda\rho\mu} + D_\mu R^{\kappa}_{\lambda\nu\rho} \\ = T^{\kappa}_{\mu\nu} R^{\lambda}_{\rho\lambda} + T^{\kappa}_{\rho\mu} R^{\lambda}_{\nu\lambda} + T^{\kappa}_{\nu\rho} R^{\lambda}_{\mu\lambda} \quad - (1)$$

Using Hodge duality, eq. (1) can be written as:

$$D_\mu \tilde{R}^a_{\lambda}{}^{\mu\nu} := R^a_{\lambda\mu\alpha} \tilde{T}^{\alpha\mu\nu} \quad - (2)$$

$$\text{i.e. } D_\mu \tilde{T}^{\alpha\mu\nu} + \omega^a_{\mu b} \tilde{T}^{\alpha\mu\nu} := \tilde{R}^a_{\mu}{}^{\mu\nu} \quad - (3)$$

The Cartan identity, is similar in structure to eq. (2), which can be written as:

$$D_\mu \tilde{R}^a_{\lambda}{}^{\mu\nu} + \omega^a_{\mu b} \tilde{R}^b_{\lambda}{}^{\mu\nu} := R^a_{\lambda\mu\alpha} \tilde{T}^{\alpha\mu\nu} \quad - (4)$$

$$\text{i.e. } D_\mu \tilde{R}^a_{\lambda}{}^{\mu\nu} := \tilde{J}^a_{\lambda}{}^{\mu\nu} \quad - (5)$$

$$\text{where: } \tilde{J}^a_{\lambda}{}^{\mu\nu} = R^a_{\lambda\mu\alpha} \tilde{T}^{\alpha\mu\nu} - \omega^a_{\mu b} \tilde{R}^b_{\lambda}{}^{\mu\nu} \quad - (6)$$

Therefore:

$$\underline{\nabla} \cdot \underline{\tilde{R}}^a_{\lambda}(\text{spin}) = \tilde{J}^a_{\lambda}{}^0 \quad - (7)$$

$$\text{and } \underline{\nabla} \times \underline{\tilde{R}}^a_{\lambda}(\text{total}) + \frac{1}{c} \frac{d}{dt} \underline{\tilde{R}}^a_{\lambda}(\text{spin}) = \underline{\tilde{J}}^a_{\lambda} \quad - (8)$$

2) where:

$$\underline{J}^a_\lambda = J^a_{x\lambda} \underline{i} + J^a_{y\lambda} \underline{j} + J^a_{z\lambda} \underline{k} \quad - (9)$$

$$\text{with: } J^a_{x\lambda} = J^{a1}_\lambda = J^{a32}_\lambda \in^1_{32} \quad - (10)$$

$$J^a_{y\lambda} = J^{a2}_\lambda = J^{a13}_\lambda \in^2_{13} \quad - (11)$$

$$J^a_{z\lambda} = J^{a3}_\lambda = J^{a21}_\lambda \in^3_{21} \quad - (12)$$

In these equations the e/n field tensor is defined by:

$$F^{\alpha}_{\mu\nu} = A^{(\alpha)} T^{\alpha}_{\mu\nu} \quad - (13)$$

and

$$\tilde{F}^{\alpha\mu\nu} = A^{(\alpha)} \tilde{T}^{\alpha\mu\nu} \quad - (14)$$

So new electrodynamical equations can be constructed.
