

2. Under tests of the Euler Theory in 2D and 3D

The EGR Lagrangian is:

$$L = \frac{1}{2} m \underline{\dot{r}} \cdot \underline{\dot{r}} + \frac{m M G}{|\underline{r}|} + \frac{M G L^2}{m c^2 |\underline{r}|^3} \quad - (1)$$

$$= \frac{1}{2} m \underline{\dot{r}} \cdot \underline{\dot{r}} + \frac{m M G}{(\underline{r} \cdot \underline{r})^{1/2}} + \frac{M G L^2}{m c^2 (\underline{r} \cdot \underline{r})^{3/2}}$$

2. ch is true for any coordinate system and in two or 3D dimensions.

If the proper Lagrange variable is chosen to be r , then:

$$\frac{dL}{dr} = \frac{d}{dt} \frac{dL}{dr} \quad - (2)$$

6 ch gives:

$$m \underline{\ddot{r}} = -m M G \frac{\underline{r}}{r^3} - \frac{3 M G L^2}{m c^2} \frac{\underline{r}}{r^5} \quad - (3)$$

e. $\underline{g} = \underline{\ddot{r}} = - M G \frac{\underline{r}}{r^3} - \frac{3 M G L^2}{m c^2} \frac{\underline{r}}{r^5} \quad - (4)$

For comparison, the same method for E(1) Theory

ves $\underline{g} = \underline{\ddot{r}} = - \frac{M G}{r^3} \frac{\underline{r}}{r^3} \quad - (5)$

Eq. (5) gives retrograde precession. Both eqs. (4) and (5) apply in 2D or 3D and in any

control system.

The numerical method used to solve eq. (5) for the orbit is previous UFT paper can now be used for eq. (4).

It is known that eq. (4) is equivalent to the Binet equation in plane polar coordinates:

$$\frac{d^2 u}{d\phi^2} + u = \frac{1}{a} + \frac{3MG}{c^2} u^2 \quad (6)$$

also

$$u = \frac{1}{r} \quad (7)$$

and

$$a = \frac{L^2}{m^2 MG} \quad (8)$$

and in UFT 391, Eq. (6) was shown to fail catastrophically. Therefore eq. (4) should also fail catastrophically when tested over a sufficient range of orbital conditions. It should fail in 2D and 3D and in any coordinate system. It is very important to display the failure graphically so that the Euler theory can be refuted completely.

The ECE2 eq. (5) on the other hand does not fail and produces retrograde precession. Eq. (5) can be used in 2D and 3D and in any coordinate system.

Two Dimensional Cartesian

Eq. (4) gives:

$$\ddot{\mathbf{x}} = -MG \left(\frac{\mathbf{x}}{(x^2 + y^2)^{3/2}} + \frac{3L^2}{m^2 c^2} \frac{\mathbf{x}}{(x^2 + y^2)^{5/2}} \right) \quad (9)$$

$$\ddot{y} = \frac{-mGy}{(x^2+y^2)^{3/2}} \left(1 + \frac{3L^2}{m^2c^2(x^2+y^2)} \right) \quad - (10)$$

The relativistic correction is eqs. (9) and (10) is:

$$\Delta(\text{rel}) = \frac{3L^2}{m^2c^2(x^2+y^2)} \quad - (11)$$

and is maximized for:

$$\frac{L^2}{(x^2+y^2)} \gg m^2c^2 \quad - (12)$$

Eqs. (9) and (10) can be integrated numerically with Runge-Kutta to give the EGR orbit. The same type of catastrophic failure should be observed as shown in Eq. (6). Eqs. (9) and (10) can also be tested to see if they give retrograde precession or forward precession.

Three Dimensional Cartesian

Eq. (4) gives three simultaneous equations:

$$\ddot{x} = \frac{-mGx}{(x^2+y^2+z^2)^{3/2}} \left(1 + \frac{3L^2}{m^2c^2(x^2+y^2+z^2)} \right) \quad - (13)$$

$$\ddot{y} = \frac{-mGy}{(x^2+y^2+z^2)^{3/2}} \left(1 + \frac{3L^2}{m^2c^2(x^2+y^2+z^2)} \right) \quad - (14)$$

$$\ddot{z} = \frac{-mGz}{(x^2+y^2+z^2)^{3/2}} \left(1 + \frac{3L^2}{m^2c^2(x^2+y^2+z^2)} \right) \quad - (15)$$

The relativistic correction is maximized for:

$$3L^2 \gg mc^2(x'^2 + y'^2 + z'^2) - (16)$$

The three dimensional orbit should feel drastically
 under the criteria (16), and can be plotted in 3-D.

Two Dimensional Plane Polar Coordinates (r, ϕ)

In this case:

$$\underline{r} = r \underline{e}_r - (17)$$

$$\underline{\ddot{r}} = (\ddot{r} - r\dot{\phi}^2) \underline{e}_r + (r\ddot{\phi} + 2\dot{r}\dot{\phi}) \underline{e}_\phi - (18)$$

which:

$$(\ddot{r} - r\dot{\phi}^2) \underline{e}_r = \frac{d^2 r}{dt^2} \underline{e}_r + \underline{\omega} \times (\underline{\omega} \times \underline{r}) - (19)$$

and:

$$(r\ddot{\phi} + 2\dot{r}\dot{\phi}) \underline{e}_\phi = \frac{d\underline{\omega}}{dt} \times \underline{r} + 2\underline{\omega} \times \underline{\dot{r}} - (20)$$

as is "Principle of ECE" volume one.

It follows that:

$$\ddot{r} - r\dot{\phi}^2 = -\frac{mg}{r} \left(1 + \frac{3L^2}{m^2 c^2 r^2} \right) - (21)$$

which is the EGR correction to the Leibniz equation.

The relativistic correction is:

$$\Delta = \frac{3L^2}{m^2 c^2 r^2} - (22)$$

It also follows that:

$$r\ddot{\phi} + 2\dot{r}\dot{\phi} = 0 - (23)$$

Eqs. (21) and (23) follow from eq. (4), in which proper Lagrange variable is chosen to be $\frac{r}{r_0}$, and Euler Lagrange equation is chosen to be eq. (2).
in plane polar coordinates:

$$\underline{\dot{r}} = \dot{r} \underline{e}_r - (24)$$

$$\underline{\dot{r}} = \dot{r} \underline{e}_r + \dot{\phi} r \underline{e}_\phi - (25)$$

is "Principle of FEE".

Using the numerical methods developed in the FT series of papers and books, eqs. (21) and (23) are solved simultaneously for $r(\phi)$. It is very likely that the solutions should fail catastrophically for $\Delta > 1$.
It can also be checked whether eqs. (21) and (23) give retrograde precession.

If the proper Lagrange variables are chosen to be r and ϕ then the two Euler Lagrange equations:

$$\frac{\partial \mathcal{L}}{\partial r} = \frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \dot{r}} - (26)$$

$$\frac{\partial \mathcal{L}}{\partial \phi} = \frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \dot{\phi}} - (27) \quad - (28)$$

$$\mathcal{L} = \frac{1}{2} m (\dot{r}^2 + \dot{\phi}^2 r^2) + \frac{m M G}{r} \left(\frac{1 + \frac{L^2}{m^2 c^2 r^2}}{\frac{1}{m c^2 r^2}} \right)$$

Eq. (27) gives:

$$\frac{d\mathcal{L}}{dt} = 0 - (29)$$

where

$L = m r^2 \dot{\phi}$ - (30)
 is the angular momentum, a constant of motion.

Eq. (2) gives:

$$m \ddot{r} = m \dot{\phi}^2 r - \frac{m G}{r^2} - \frac{3 m G L^2}{m c^2 r^4} \quad - (31)$$

i.e. $\ddot{r} = \dot{\phi}^2 r - \frac{m G}{r^2} \left(1 + \frac{3 L^2}{m c^2 r^2} \right) \quad - (32)$

Eqs (21) and (32) are the same, Q.E.D.

From eqs. (30) and (32):

$$\ddot{r} = \frac{L^2}{m^2 r^3} - \frac{m G}{r^2} \left(1 + \frac{3 L^2}{m c^2 r^2} \right) \quad - (33)$$

which is the EBR corrected Newton equation.

Note carefully that eq. (2) in plane polar coordinates has the advantage of giving eq. (23):

$$r \ddot{\phi} + 2 \dot{r} \dot{\phi} = 0 \quad - (34)$$

Eqs (33) and (34) can be integrated simultaneously to give the orbit $r(\phi)$. Under the condition $\Delta \gg 1$, r should feel drastically.

Finally, the problem of solving eq. (1) for the orbit can also be solved using X, Y and Z as the Lagrange variables. This will be described in the next note.