

433(1): Methods of Quantization of the String Field

In a theory, the string field is defined by the force magnitude:

$$F = - \left(\frac{dm(r)}{dr} \frac{m^{1/2}(r)}{2m(r) - r \frac{dm(r)}{dr}} \right) \bar{E} \quad (1)$$

where

$$\bar{E}^2 = p^2 c^2 + m(r) m^2 c^4 \quad (2)$$

is an expression for the energy of the field. Eq. (1) can represent the string force between neutrons and protons or the string force between quarks inside a proton or neutron.

Eq. (2) can be quantized into a wave equation as in HFT 431 using:

$$E = i\hbar \frac{\partial}{\partial t}, \quad \underline{p} = -i\hbar \underline{\nabla} \quad (3)$$

to obtain:

$$\left(\square + m(r) \left(\frac{mc}{\hbar} \right)^2 \right) \phi = 0 \quad (4)$$

where

$$m(r) = - \frac{\hbar}{mc} \gamma_a^{\mu\nu} \partial_\mu \left(\omega_{\mu\nu}^a - \pi_{\mu\nu}^a \right) \quad (5)$$

Eq. (4) was developed in HFT 431, where it was shown that ϕ has features reminiscent of the Bessel equation.

It is immediately clear from eq. (4) that the pion waves are governed by $m(r)$ and ϕ . There are three pions, so eq. (4) should give three waves. The choice of $m(r)$ and ϕ .

Second quantization occurs through the de Broglie-Bohr equation:

$$E^2 = p^2 c^2 + m(r) m^2 c^4 = \hbar^2 \omega^2 \quad - (6)$$

and because of the fact that the pion travels at close to the speed of light:

$$E^2 = \hbar^2 \kappa^2 c^2 + m(r) m^2 c^4 = \hbar^2 \omega^2 \quad - (7)$$

with

$$\omega \sim \kappa c \quad - (8)$$

so

$$E = \hbar \omega \quad - (9)$$

to a good approximation. Eq. (9) is always true provided that

$$\hbar^2 \omega^2 = \hbar^2 \kappa^2 c^2 + m(r) m^2 c^4 \quad - (10)$$

i.e.

$$\omega^2 = \kappa^2 c^2 + m(r) \frac{m^2 c^4}{\hbar^2} \quad - (11)$$

The energy levels of the field can be obtained by writing Eq. (2) as:

$$E^2 - m(r) m^2 c^4 = p^2 c^2 \quad - (12)$$

$$\therefore e \quad (E - m(r)^{1/2} m c^2) (E + m(r)^{1/2} m c^2) = p^2 c^2 \quad - (13)$$

$$\text{so} \quad E = \frac{p^2 c^2}{E + m(r)^{1/2} m c^2} + m(r)^{1/2} m c^2 \quad - (14)$$

The usual Dirac theory assumes that E in the denominator of Eq. (14) can be approximated by the rest energy

$$E_0 = m(r)^{1/2} m c^2 \quad - (15)$$

but if the pion travels close to the speed of light this approximation can no longer be used.
However, from eqs. (9) and (14):

$$E = \frac{p^2 c^2}{2\omega + m(r)^{1/2} m c^2} + m(r)^{1/2} m c^2 \quad (16)$$

so

$$E\psi = \frac{-\hbar^2 c^2 \nabla^2 \psi}{2\omega + m(r)^{1/2} m c^2} + m(r)^{1/2} m c^2 \psi \quad (17)$$

From Eq. (12):

$$(2\omega)^2 = p^2 c^2 + m(r) m^2 c^4 \quad (18)$$

and using the quantization:

$$\underline{p} \psi = -i\hbar \underline{\nabla} \psi \quad (19)$$

$$- (20)$$

eq. (18) quantizes to:

$$(2\omega)^2 = -\hbar^2 c^2 \int \psi^* \nabla^2 \psi d\tau + m^2 c^4 \int \psi^* m(r) \psi d\tau$$

so:

$$m^2 c^4 = \frac{(2\omega)^2 + \hbar^2 c^2 \int \psi^* \nabla^2 \psi d\tau}{\int \psi^* m(r) \psi d\tau} \quad (21)$$

Therefore the masses of the pions are given by the equation:

$$m^2 = \frac{1}{c^4} \left(\frac{(\hbar\omega)^2 + \hbar^2 c^2 \int \psi^* \nabla^2 \psi d\tau}{\int \psi^* m(r) \psi d\tau} \right) - (22)$$

The mass of the rest pions are given by:

$$m^2 = \frac{1}{c^4} \left(\frac{(\hbar\omega)^2}{\int \psi^* m(r) \psi d\tau} \right) - (23)$$

and in Minkowski spacetime:

$$\hbar\omega = mc^2 - (24)$$

which is the de Broglie rest mass equation, because in Minkowski spacetime:

$$m(r) = 1 - (25)$$

and

$$\int \psi^* \psi d\tau = 1. - (26)$$